12.8 - 12.12 Review

Simplify each number by using the imaginary number *i*.

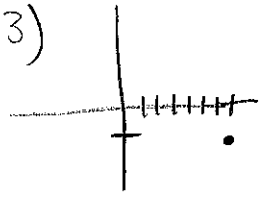
1. $\sqrt{-48}$ → $i\sqrt{48}$

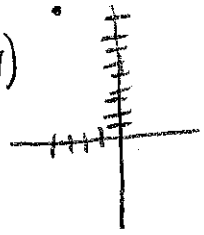
2. $\sqrt{-36}$ → $6i$

Plot each complex number and find its absolute value.

3. $7 - 1i$ → $\sqrt{7^2 + 1^2} = \sqrt{50}$

4. $-4 + 9i$ → $\sqrt{4^2 + 9^2} = \sqrt{97}$

3) 

4) 

Simplify each expression.

5. $(9 + 6i) + (2 - i)$ → $11 + 5i$

6. $(-12i) - (3 + 3i)$ → $-3 - 15i$

7. $(-2i)(5 + 4i)$

$-10i - 8i^2 = -10i - 8(-1)$

$= 8 - 10i$

Write each quotient as a complex number.

8. $\frac{5 + 4i}{0 + 7i}\left(\frac{0 - 7i}{0 - 7i}\right)$

	5	4i
0	0	0
-7i	-35i	$-28i^2$

$-35i - 28i^{2(-1)}$

$28 - 35i$

	0	7i
0	0	0
-7i	0	$-49i^2$

$-49i^{2(-1)}$

49

$\frac{28 - 35i}{49}$

$= \frac{4 - 5i}{7}$

$= \frac{4}{7} - \frac{5i}{7}$

9. $\frac{-1 + 5i}{3 - 2i}\left(\frac{3 + 2i}{3 + 2i}\right)$

	-1	5i
3	-3	15i
2i	-2i	$10i^2$

$-3 + 13i + 10i^{(-1)}$

$-3 + 13i - 10$

$-13 + 13i$

	3	-2i
3	9	-6i
2i	6i	$-4i^2$

$9 - 4i^{2(-1)}$

$9 + 4 = 13$

$\frac{-13 + 13i}{13} = \frac{-1 + i}{1}$

$= -1 + i$

Find all solutions to each quadratic equation.

10. $2x^2 - 3x + 7 = 0$ (a: 2, b: −3, c: 7)

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(7)}}{2(2)}$

$= \frac{3 \pm \sqrt{-47}}{4} = \frac{3 \pm i\sqrt{47}}{4}$

11. $4x^2 - 5x + 6 = 0$ (a: 4, b: −5, c: 6)

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(6)}}{2(4)}$

$= \frac{5 \pm \sqrt{-71}}{8}$

$= \frac{5 \pm i\sqrt{71}}{8}$

12. $x(x - 3) + 3 = 0$

$x^2 - 3x + 3 = 0$ (a: 1, b: −3, c: 3)

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)}$

$= \frac{3 \pm \sqrt{-3}}{2} = \frac{3 \pm i\sqrt{3}}{2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

13. **Error Analysis** Robert solved the equation $2x^2 + 16 = 0$. His solution was $x = \pm\sqrt{-8}\,i$. What errors did Robert make? What is the correct solution?

Should have taken out negative

$2x^2 + 16 = 0$

$-16 \quad -16$

$\frac{2x^2}{2} = \frac{-16}{2}$

$\sqrt{x^2} = \sqrt{-8}$

$x = \pm\sqrt{-8} = \pm i\sqrt{8}$

Solve each system

14. $y = x^2$, $y = 3x$

$x^2 = 3x$, $-3x \quad -3x$, $x^2 - 3x = 0$, $x(x-3) = 0$

$x = 0$, $y = 0^2 = 0$, $(0,0)$

$x - 3 = 0$, $+3 \quad +3$, $x = 3$, $y = 3^2 = 9$, $(3,9)$

16. $y = x^2 + 6x - 8$, $y = x - 2$

$x - 2 = x^2 + 6x - 8$, $-x \quad +2 \quad -x \quad +2$, $0 = x^2 + 5x - 6$, $0 = (x+6)(x-1)$

$x + 6 = 0$, $-6 \quad -6$, $x = -6$, $y = -6 - 2 = -8$, $(-6, -8)$

$x - 1 = 0$, $+1 \quad +1$, $x = 1$, $y = 1 - 2 = -1$, $(1, -1)$

15. $y = x^2 - 5x$, $y = x - 8$

$x - 10 = x^2 + 20x + 80$, $-x \quad +10 \quad -x \quad +10$, $0 = x^2 + 19x + 90$, $0 = (x+10)(x+9)$

$x + 10 = 0$, $-10 \quad -10$, $x = -10$, $y = -10 - 10 = -20$, $(-10, -20)$

$x + 9 = 0$, $-9 \quad -9$, $x = -9$, $y = -9 - 10 = -19$, $(-9, -19)$

17. $y = x^2 + 20x + 80$, $y = x - 10$

$x - 2 = x^2 + 6x - 8$, $-x \quad +2$, $0 = x^2 + 5x - 6$, $0 = (x+6)(x-1)$

$x + 6 = 0$, $-6 \quad -6$, $x = -6$, $y = -6 - 2 = -8$, $(-6, -8)$

$x - 1 = 0$, $+1 \quad +1$, $x = 1$, $y = 1 - 2 = -1$, $(1, -1)$

18. The sales of two different products are modeled by the equations shown below. The sales are represented by y and the number of weeks the products have been selling is represented by x. According to the projections, what week(s) did the products have the same amount of sales? What were the sales of both products during the week(s) of equal sales?

Product 1: $y = x^2 - 17x + 89$

Product 2: $y = 17x + 25$

$17x + 25 = x^2 - 17x + 89$, $-17x \quad -25 \quad -17x \quad -25$, $0 = x^2 - 34x + 64$, $0 = (x-32)(x-2)$

$x - 32 = 0$, $+32 \quad +32$, $x = 32$, $y = 17(32) + 25 = 569$

$x - 2 = 0$, $+2 \quad +2$, $x = 2$, $y = 17(2) + 25 = 69$

Week 32 → $569
Week 2 → $69

19. The population of two different villages are modeled by the equations shown below. The population (in thousands) is represented by y and the number of years since 1975 is represented by x. What year(s) did the villages have the same population? What was the population of both cities during the year(s) of equal population?

Lewiston: $y = x^2 - 30x + 540$

Lockport: $y = 20x + 15$

$20x + 15 = x^2 - 30x + 540$, $-20x \quad -15 \quad -20x \quad -15$, $0 = x^2 - 50x + 525$, $0 = (x-35)(x-15)$

$x - 35 = 0$, $+35 \quad +35$, $x = 35 \rightarrow 1975 + 35 = 2010$, $y = 20(35) + 15 = 715$

$x - 15 = 0$, $+15 \quad +15$, $x = 15 \rightarrow 1975 + 15 = 1990$, $y = 20(15) + 15 = 315$

In 1990 → 315 people
In 2010 → 715 people

Write an equation of a circle with the given center and radius. Check your answers. $(x-h)^2 + (y-k)^2 = r^2$

20. center $(5, -3)$, radius 1

$(x-5)^2 + (y-(-3))^2 = 1^2$

$(x-5)^2 + (y+3)^2 = 1$

21. center $(-1, -5)$, radius 5

$(x-(-1))^2 + (y-(-5))^2 = 5^2$

$(x+1)^2 + (y+5)^2 = 25$

Write an equation for each translation.

22. $x^2 + y^2 = 81$; right 4 units

$(x-4)^2 + y^2 = 81$

23. $x^2 + y^2 = 1$; left 1 unit and up 1 unit

$(x+1)^2 + (y-1)^2 = 1$

24. $x^2 + y^2 = 4$; up 7 units

$x^2 + (y-7)^2 = 4$

25. $x^2 + y^2 = 36$; right 5 units and down 2 units

$(x-5)^2 + (y+2)^2 = 36$

For each equation, find the center and radius of the circle.

26. $(x+2)^2 + (y+2)^2 = 64$ ($-h$, $-k$, r^2)

Center = $(-2, -2)$
radius = $\sqrt{64} = 8$

27. $x^2 + (y-5)^2 = 16$ ($(x-h)^2$, k, r^2)

Center = $(0, 5)$
radius = $\sqrt{16} = 4$

Use the given information to write an equation of the circle.

28. center $(-3, 1)$, through $(-3, -2)$ (h, k, $r = 3$)

$r = \sqrt{(-3-(-3))^2 + (-2-1)^2} = \sqrt{9} = 3$

$(x+3)^2 + (y-1)^2 = 9$